MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2018/2019

EMT2046 – ENGINEERING MATHEMATICS 4

(All Sections / Groups)

29 MAY 2019

9:00 AM – 11:00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **9 pages** (including cover page) with **4** Questions only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) Consider the function $f(x)=3+2\ln(x)$.

(i) Approximate the integral $\int_1^7 f(x)dx$ by using the composite Simpson's rule with 2 intervals.

[5 marks]

(ii) Calculate the absolute error for Question 1(a)(i).
[Hint: $\int \ln(x)dx = x\ln(x)-x+C$]

[4 marks]

(iii) Compute the error bound for Question 1(a)(i).

[4 marks]

(b) Consider the following system of linear equations,

$$7x_1-2x_2+x_3+2x_4=3$$
$$2x_1+8x_2+3x_3+x_4=-2$$
$$-x_1+5x_3+2x_4=5$$
$$2x_2-x_3+4x_4=4$$

By using Gauss-Seidel technique, estimate $[x_1,x_2,x_3,x_4]$ after two iterations starting from $[x_1,x_2,x_3,x_4]=[0,-1,1,1]$.

[12 marks]

Continued...

Question 2

(a) If the joint probability function of X and Y is given by

$$f_{XY}(x,y)=\begin{cases}k(x+2y), & 0<x<1,1<y<2,\\ 0, & \text{otherwise.}\end{cases}$$

where k is a constant.

(i) What is the value of k? [5 marks]

(ii) Find the expected value of XY. [5 marks]

(b) Consider the Markov chain with state space $\{0,1,2\}$ and transition probability matrix as below:

$$\begin{bmatrix} a & 0.3 & b \\ a-b & 0.2 & 0.6 \\ 0 & 0.8 & 0.2 \end{bmatrix}$$

where a and b are constants.

(i) What are the values of a and b? [3 marks]

(ii) Draw the state transition diagram. [4 marks]

(iii) Find the stationary probability vector $[\pi_0 \quad \pi_1 \quad \pi_2]$. [8 marks]

Continued...

Question 3

(a) Consider the following linear programming (LP) problem:

$$\begin{aligned} \text{Maximize } z = \; & 3x_1 + 5x_2 + x_3 \\ \text{subject to:} \quad & 2x_1 + x_2 + 2x_3 \le 20 \quad \text{(Constraint 1)} \\ & x_1 + 3x_2 + x_3 \le 30 \quad \text{(Constraint 2)} \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Express the LP problem in standard form and solve it using the **simplex method**. Remember to specify the **entering and leaving variables**, as well as the elementary row operations for each iteration. For the benefit of comparison, a partially-filled simplex tableau at the optimal stage is shown in Table Q3a, with s_1 and s_2 are the slack variables for Constraints 1 and 2, respectively.

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	0	0	2	$\frac{4}{5}$	$\frac{7}{5}$	58
?	0	?	?	?	?	?	6
?	0	?	?	?	?	?	8

Table Q3a

[15 marks]

(b) Convert the above LP problem to its dual.

[5 marks]

(c) Using appropriate properties, deduce the optimal solution and optimal value for the dual problem in (b).

[3 marks]

(d) What would happen to the optimal value of the primal problem if:

(i) the right-hand-side (RHS) of its Constraint 1 is decreased by 1 unit?

(ii) the RHS of its Constraint 2 is increased by 1 unit?

[2 marks]

Continued...

Question 4

(a) A transportation company will soon begin providing bus service to connect town A to town F. Figure Q4a outlines different possible routes that connect town A to town F. The nodes represent the towns, and each number along a link represents the distance connected by that link.

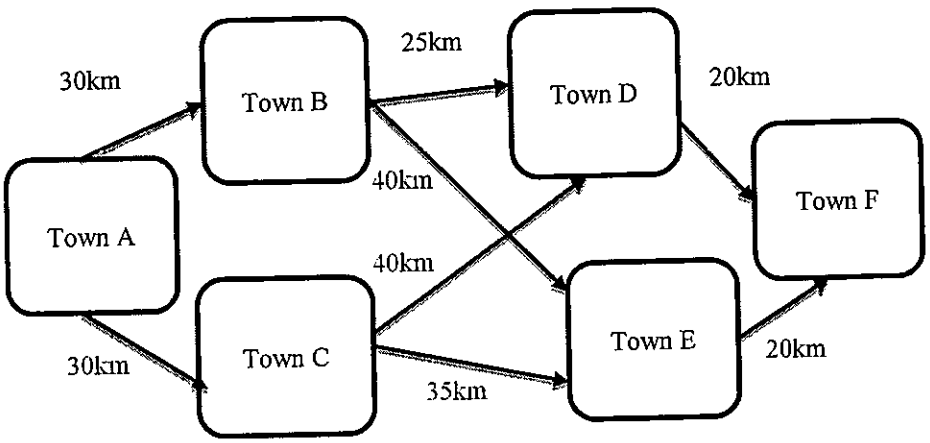

Figure Q4a

The problem is to determine which connecting bus routes should be used to connect town A to town F with a minimum total length of distance. Use dynamic Programming (DP) to solve the problem. In your solution, you must:

1. Define clearly what the stages, states, recursive formulae and decision variables of your DP model represent.
2. Indicate the applicable states for each stage and show necessary computations for the recursive formulae in obtaining the decision variable.

Finally, state the optimal policy (route) and the corresponding optimal distance.

[13 marks]

Continued...

Question 4

(b) Find the minimal spanning tree of the graph in Figure Q4b using **Kruskal's algorithm**.

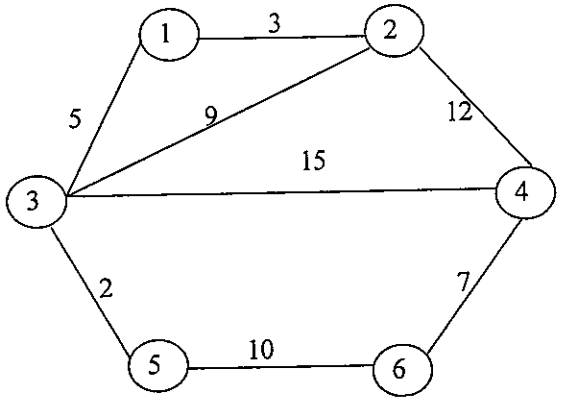

Figure Q4b

Specify clearly which edges are added or rejected during intermediate iterations/steps. Draw the resulting network at the termination of the algorithm.

[12 marks]

Continued...

APPENDIX I: List of Formulas

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0) \cdots (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a)+f(b)\right).$$

The error term is $\left|\frac{h^3 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = b - a$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right].$$

The error term is $\left|\frac{(b-a)h^2 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = \frac{b-a}{n}$.

10. Simpson's rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right].$$

The error term is $\left|\frac{h^5}{90} f^{(iv)}(\xi)\right|$ for some ξ in (a, b) and $h = \frac{b-a}{2}$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right].$$

The error term is $\left|\frac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$ for some ξ in (a, b), $h = \frac{b-a}{n}$,

$x_j = a + jh$, $j = 0, 1, \ldots, n$.

Continued...

12. Four-term Gaussian Quadrature

$$\int_a^b f(x)\,dx = \frac{(b-a)}{2}\int_{-1}^{1} f\left(x = \frac{(b+a)+t(b-a)}{2}\right)dt$$

$$\int_{-1}^{1} f(x)\,dx \approx w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) + w_4 f(x_4)$$

Number of terms n	Nodes x_i	Weights w_i
4	-0.8611363116	0.3478548451
	-0.3399810436	0.6521451549
	0.3399810436	0.6521451549
	0.8611363116	0.3478548451

13. Newton-Raphson's method

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, \qquad n = 1,2,...$$

14. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2}Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

15. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

16. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1),$$
$$k_3 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

End of paper.